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The effect of probabilities of departure with time in a bank

Kasturi Nirmala, Shahnaz Bathul

Abstract— This paper deals with the Queueing theory and analysis of probability curves of pure death model. Starting with the historical back grounds and important concepts of Queueing theory, we obtained a relation to find time "t" where we get highest probability for departures which follow truncated Poisson probability distribution.

Index Terms— Arrivals allowed at initial time, arrival rate, departure rate, highest probability, Queues, probability curves, truncated Poisson probability distribution

1 INTRODUCTION

A lot of our time is consumed by unproductive activities. Travelling has its own demerits one of them being wastage of time getting caught in traffic jams. A visit to the Post office or bank is very time consuming as huge crowds are waiting to be serviced. A simple shopping chore to a supermarket leads us to face long queues. In general we do not like to wait.

People who are giving us service also do not like these delays because of loss of their business. These waits are happening due to the lack of service facility. To provide a solution to these problems we analyze queueing systems to understand the size of the queue, behavior of the customers in the queue, system capacity, arrival process, service availability, service process in the system. After analyzing the queueing system we can give suggestions to management to take good decisions.

A queue is a waiting line. Queueing theory is mathematical theory of waiting lines. The customers arriving at a queue may be calls, messages, persons, machines, tasks etc. we identify the unit demanding service, whether it is human or otherwise, as a customer. The unit providing service is known as server. For example (1) vehicles requiring service wait for their turn in a service center. (2) Patients arrive at a hospital for treatment. (3) Shoppers are face with long billing queues in super markets. (4) Passengers exhaust a lot of time from the time they enter the airport starting with baggage, security checks and boarding.

Queueing theory studies arrival process in to the system, waiting time in the queue, waiting time in the system and service process. And in general we observe the following type of behavior with the customer in the queue. They are

Balking of Queue: Some customers decide not to join the queue due to their observation related to the long length of queue, in sufficient waiting space. This is called Balking.

Reneging of Queue: This is the about impatient customers. Customers after being in queue for some time, few customers become impatient and may leave the queue. This phenomenon is called as Reneging of Queue.

Jockeying of Queue: Jockeying is a phenomenon in which the customers move from one queue to another queue with hope that they will receive quicker service in the new position.

Important concepts in Queueing theory

Little law

One of the feet of queueing theory is the formula Little law. This is

$$N=\lambda T$$

This formula applies to any system in equilibrium (steady state).

Let λ is the arrival rate

T is the average time a customer spends in the system

N is the average number of customers in the system

Little law can be applied to the queue itself.

I.e. $$N_q = \lambda T_q$$

Where λ is the arrival rate

T_q the average time a customer spends in the queue

N_q is the average number of customers in the queue

Classification of queuing systems

Input process

If the occurrence of arrivals and the offer of service strictly follow some schedule, a queue can be avoided. In practice this is not possible for all systems. Therefore the best way to describe the input process is by using random variables which we can define as "Number of arrivals during the time interval" or "The time interval between successive arrivals"

Service Process

Random variables are used to describe the service process which we can define as "service time" or "no of servers" when necessary.

Number of servers

Single or multiple servers

Queue length

1 to ∞

System capacity

- *Kasturi Nirmala is currently pursuing doctoral degree program in department of Mathematics in Jawaharlal Nehru Technological University, Hyderabad, India, PH-91-9703046412. E-mail: vaka_nirmala@yahoo.com*
- *Dr. Shahnaz Bathul is a Professor at department of Mathematics in Jawaharlal Nehru Technological University, Hyderabad, India, PH-91- 9948491418. E-mail: shahnazbathul@yahoo.com*

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Finite or Infinite

Queue discipline

This is the rule followed by server in accepting the customers to give service. The rules are

- FCFS (First come first served).
- LCFS (Last come first served).
- Random selection (RS).
- Priority will be given to some customers.
- General discipline (GD).

Notation for describing all characteristics above of a queueing model was first suggested by David G Kendall in 1953.

- A/B/P/Q/R/Z

Where A indicates the distribution of inter arrival times
B denotes the distribution of the service times
P is the number of servers
Q is the capacity of the system
R denotes number of sources
Z refers to the service discipline

Examples of queueing systems that can be defined with this convention are M/M/1
M/D/n
G/G/n

Where M stands for Markov
D stands for deterministic
G stands for general

History In Telephone system we provide communication paths between pairs of customers on demand. The permanent communication path between two telephone sets would be expensive and impossible. So to build a communication path between a pair of customers, the telephone sets are provided a common pool, which is used by telephone set whenever required and returns back to pool after completing the call. So automatically calls experience delays when the server is busy. To reduce the delay we have to provide sufficient equipment. To study how much equipment must be provided to reduce the delay we have to analyze queue at the pool. In 1908 Copenhagen Telephone Company requested Agner K.Erlang to work on the holding times in a telephone switch. Erlang's task can be formulated as follows. What fraction of the incoming calls is lost because of the busy line at the telephone exchange? First we should know the inter arrival and service time distributions. After collecting data, Erlang verified that the Poisson process arrivals and exponentially distributed service were appropriate mathematical assumptions. He had found steady state probability that an arriving call is lost and the steady state probability that an arriving customer has to wait. Assuming that arrival rate is λ, service rate is μ and $\rho = \lambda/\mu$ he derived formulae for loss and deley.

(1) The probability that an arriving call is lost (which is known as Erlang B-formula or loss formula).

$$P_n = \frac{\frac{\rho}{n!}}{\sum \frac{\rho^k}{k!}} = B(n,\rho)$$

(2) The probability that an arriving has to wait (which is known as Erlang C-formula or deley formula).

$$P_n = \frac{n}{n-\rho(1-B(n,\rho))} B(n,\rho)$$

Erlang's paper "On the rational determination of number of circuits" deals with the calculation of the optimum number of channels so as to reduce the probability of loss in the system. Whole theory started with a congestion problem in tele-traffic. The application of queueing theory scattered many areas. It include not only tele-communications but also traffic control, hospitals, military, call-centers, supermarkets, computer science, engineering, management science and many other areas.

Truncated Poisson probability distribution

In pure death model, the system starts with N customers at time 0 and no other arrivals are allowed. In a queueing system if the departures occur at the rate μ customers per unit time t then the probability of n customers remaining after time t units is

$$P_n(t) = \frac{\mu(t)^{N-n} e^{-\mu t}}{(N-n)!}$$

$$P_0(t) = 1 - \sum_{n=1}^{N} Pn(t)$$

This is the truncated Poisson probability distribution.

t is used to define the interval 0 to t.
μ is the total average departure rate in departures per second.
N is the number of customers allowed at time t=0 seconds.
$P_0(t)$ is probability of zero customers remained at time t seconds.
$P_n(t)$ is probability of n customers remained at time t seconds.

A short survey conducted on Andhra Bank, J.N.T.U.H. and Hyderabad. The bank has an average of 51 customers departing every 100 seconds (0.51departures/second departure rate). Let X is the Random variable defined as "Number of departures at the bank during the time interval 0 to t" which describes the output process at the bank. All the customers are independent. The probability distribution of number of departures in a fixed time interval follows a Truncated Poisson probability distribution.

$$P_n(t) = \frac{\mu(t)^{N-n} e^{-\mu t}}{(N-n)!}$$

$$P_0(t) = 1 - \sum_{n=1}^{N} Pn(t)$$

With this departure rate 0.51departures/sec and truncated Poisson probability distribution we find probability of number of departures during the time interval 0 to t. Let us assume

N = 6 i.e. initially 6 customers are allowed, other arrivals are not allowed. Then probability of departing one person is nothing but probability of 5 customers remained in the system.

ISSN 2229-5518

We calculate this by the formula

$$P_5(t) = \frac{0.51(t)^{6-5} e^{-0.51(t)}}{(6-5)!}$$

Noting the probabilities of departing 2, 3 and 4 persons during the time interval 0 to t by using the formulae

$$P_4(t) = \frac{[0.51(t)]^{6-4} e^{-0.51(t)}}{(6-4)!}$$

$$P_3(t) = \frac{[0.51(t)]^{6-3} e^{-0.51(t)}}{(6-3)!}$$

$$P_2(t) = \frac{[0.51(t)]^{6-2} e^{-0.51(t)}}{(6-2)!}$$

$$P_1(t) = \frac{[0.51(t)]^{6-1} e^{-0.51(t)}}{(6-1)!}$$

If we observe the bank for a period of 1 second, the probability of departing one customer in 0 to 1 seconds time interval is 0.306252744. If we observe the bank for 2 seconds, the probability of departing one customer in 0 to 2 seconds time interval is 0.367806838. If we observe the bank for 5 seconds the probability of departing one customer in 0 to 5 seconds time interval is 0.199108248. Up to 20 seconds the probabilities of departing n persons is calculated.

$P_5(1) = 0.306252744$ $P_4(1) = 0.078094449$
$P_5(2) = 0.367806838$ $P_4(2) = 0.187581487$
$P_5(4) = 0.26525857$ $P_4(4) = 0.270563741$
$P_5(5) = 0.19910824$ $P_4(5) = 0.253863016$
$P_2(1) = 0.00169269$ $P_1(1) = 0.00017265$
$P_2(2) = 0.01626331$ $P_1(2) = 0.00331771$
$P_2(3) = 0.04944062$ $P_1(3) = 0.01512883$
$P_2(4) = 0.093831505$ $P_1(4) = 0.03828325$
$P_2(5) = 0.137562022$ $P_1(5) = 0.07015663$
$P_3(1) = 0.013276056$
$P_3(2) = 0.063777705$
$P_3(3) = 0.129256529$
$P_3(4) = 0.183983344$
$P_3(5) = 0.215783565$

Remaining values are tabulated.

Table-1 to Table-5 gives statistics of probabilities of arrivals of n number of persons at time t.

1:

time	$P_5(t)$	time	$P_5(t)$
1	0.306252744	6	0.143476347
2	0.367806838	7	0.100516397
3	0.331299571	8	0.068982459
4	0.26525857	9	0.046601619
5	0.19910824	10	0.000379137

2:

time	$P_4(t)$	time	$P_4(t)$
1	0.078094449	11	0.05761077
2	0.187581487	12	0.041170924
3	0.253444171	13	0.029015138
4	0.270563741	14	0.020207092
5	0.253863016	15	0.013929646
6	0.219518811	16	0.009517159538
7	0.179421769	17	0.00645171302
8	0.140724218	18	0.00434342196
9	0.1069950717	19	0.00290605633
10	0.079288189	20	0.00193359997

3

time	$P_3(t)$	time	$P_3(t)$
1	0.215783565	11	0.10773214
2	0.063777705	12	0.08398868
3	0.129256529	13	0.06412345
4	0.183983344	14	0.04809287
5	0.215783565	15	0.03552059
6	0.22390918	16	0.02588667
7	0.21351190	17	0.01864545
8	0.19138493	18	0.01329087
9	0.16363459	19	0.00938656
10	0.13478992	20	0.00657423

4:

time	$P_2(t)$	time	$P_2(t)$
1	0.00169269	11	0.15109432
2	0.01626331	12	0.12850268

ISSN 2229-5518

3 0.04944062	13 0.10628462
4 0.093831505	14 0.08584579
5 0.137562022	15 0.05280881
6 0.17129052	16 0.05280881
7 0.19055937	17 0.04041401
8 0.19521263	18 0.03050254
9 0.18777070	19 0.02738946
10 0.17185714	20 0.01676431

5:

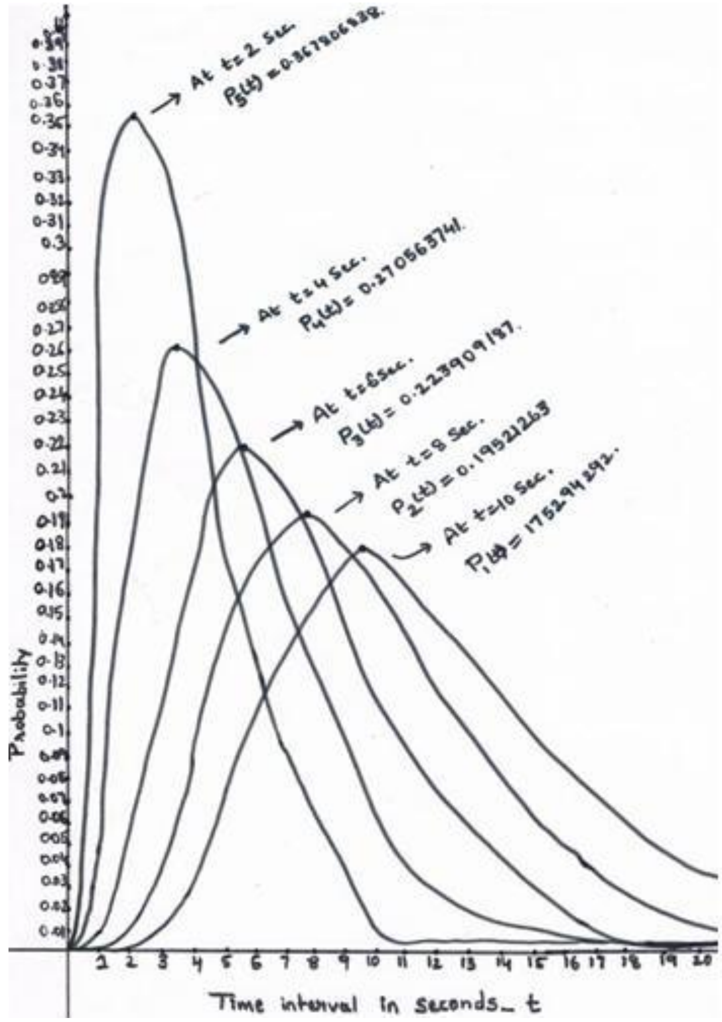
time $P_1(t)$	time $P_1(t)$
1 0.00017265	11 0.169527835
2 0.00331771	12 0.159287292
3 0.01512883	13 0.140933416
4 0.03828325	14 0.122587789
5 0.07015663	15 0.103937709
6 0.10482980	16 0.086183985
7 0.13605939	17 0.0700779
8 0.15929350	18 0.05600268
9 0.17237350	19 0.044064518
10 0.17529429	20 0.034199196

Graph (Probability graph)

Probability that number of customers departure during a specified interval.

Departure rate = 51 customers departing every 100 seconds.
Scale: Time interval in seconds (0 to 20seconds)
$P_n(t)$ is 0 to 1
Departure rate = 0.51departures/second.

A graph is drawn between time and probability of n departures at time t. Time in seconds is taken on the horizontal line, $P_n(t)$ is taken on the vertical line. Curve-1 (c-1) is derived by taking N = 6 and n = 1 in the truncated Poisson probability distribution. As well remaining curves c-2, c-3, c-4, c-5 have been derived by taking n= 2, n = 3, n = 4, n = 5 and N = 6 respectively. By observing the graph, we would note that each curve peaks at a particular point and then they start to descend. After descending to a certain point



they run parallel to the time axis. We can observe that highest probability for curve-5 is at t = 2 seconds. This indicates probability of departure of one person from the bank from 0 to 2 seconds is high. Let α denote the number of customers taken their service. Hear α= 1. Curve-4 got highest point at t = 4 seconds. Third curve got highest point at time t = 6 seconds. Curve-2 got highest point at time t = 8 seconds. First curve got highest point at t = 10 seconds
For the curve-1 at t = 10 seconds, the crest point P_1 (10) = 0.175294292.
For the curve-2 at t = 8 seconds, the crest point P_2 (8) = 0.195212635.
For the curve-3 at t = 6 seconds, the crest point P_3 (6) = 0.223909187.
For the curve-4 at t = 4 seconds the crest point P_4 (4) = 0.270563741.
For the curve-5 at t =2 seconds, the crest point P_5 (2) = 0.367806838.
Developing appropriate formula for t at which we get highest probability by use of crest points. Out of N = 6 customers, if 1 customer had taken the service, then number of people remained in the system is equal to 5. So we write n = 5 for which 1 customer

had taken service and 5 customers remained in the system.

For n = 5 we have crest point at t = 2 seconds.

t = 2(1) seconds.

t = 2(number of customer who received the service) seconds.

t = 2(α) seconds.

That is α=1 (where α is number of customers who received the service).

For n = 4, we have crest point at t = 4 seconds.

t = 2(number of customers who received the service) seconds.

t = 2(α) seconds.

For n = 3, we have crest point at t = 6 seconds.

t = 2(3) seconds.

t = 2(number of customers who received the service) seconds.

t = 2(α) seconds.

For n = 2, we have crest point at t = 8 seconds.

t = 2(4) seconds

t = 2 (number of customers who received the service) seconds.

t = 2(α) seconds.

For n = 1, we have crest point at t = 10 seconds.

t = 2(5) seconds.

t = 2(number of customers who received the service) seconds.

t = 2(α) seconds.

The above calculations can be generalized and is indicated by the following formula.

For the departures in a queueing system which follow truncated Poisson probability distribution, t = 2(N-n) seconds is the time where we get maximum probability for the curves c-1, c-2, c-3 etc and N, n are positive integers.

Conclusions

This paper studies the Queueing theory and by the use of queuing theory analysis of probability curves of pure death model. From the work surveyed above we have found departure rate at the bank is 0.51 departures per second. We Considered the Random variable as "Number of departures during the time interval 0 to t" which follows truncated Poisson probability distribution. We calculated probabilities of departures of n = 1, 2, 3, 4, 5 persons during time interval 0 to t. A graph was drawn between the time and probabilities of n departures where n = 1, 2, 3, 4, 5 from which curves c-1, c-2, c-3, c-4 and c-5 were obtained. By observing the graph, we would note that each curve peaks at particular point and then they starts to descend. That is each curve had peak point at particular time. From the above calculations we developed an equation to find this particular time t where curve attain its crest point. The equation is (Time) t = 2(N-n) seconds where N is the number of customers allowed at initial time 0 seconds, n is the number of customers remained in the system after taking N-n people service. This equation assists us in deriving time where curve takes crest point.

We observe that if n increases, P n (t) increases. That is n ∞ P n (t). If t increases P n (t) decreases.

That is $t \propto \frac{1}{Pn(t)}$.

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